Collective Modes and f-wave Pairing Interactions in Superfluid ^{3}He

J.P. Davis, H. Choi, J. Pollanen, and W.P. Halperin
Department of Physics and Astronomy,
Northwestern University, Evanston, Illinois 60208
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Precision measurements of collective mode frequencies in superfluid ^{3}He-B are sensitive to quasiparticle and f-wave pairing interactions. Measurements were performed at various pressures using interference of transverse sound in an acoustic cavity. We fit the measured collective mode frequencies, which depend on the strength of f-wave pairing and the Fermi liquid parameter F_2^s, to theoretical predictions and discuss what implications these values have for observing new order parameter collective modes.

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One of the fundamental notions of modern condensed matter physics is that of spontaneous symmetry breaking at a phase transition. A by-product of broken symmetry is the appearance of order parameter collective modes, which are a fingerprint of the underlying structure of the ordered state [1, 2]. We can determine a wealth of information about the system through measurement of these collective modes. Early observations of order parameter collective modes in superfluid ^{3}He provided conclusive confirmation of its pairing state. In addition to better understanding the superfluid we can also learn about the normal state quasi-particle interactions. The superfluid order parameter has 9 complex valued components, resulting in a rich spectrum of modes characteristic of the unconventional nature of superfluidity in ^{3}He. Similarly, one expects collective modes in other unconventional pairing systems. In this work we report precise measurements of the frequency of a collective mode in superfluid ^{3}He-B which depends, in part, on the strength of sub-dominant f-wave pairing interactions.

In addition to the known p-wave pairing that defines the equilibrium state of superfluid ^{3}He there may be nontrivial contributions from f-wave interactions [1] that appear in the dynamics of the order parameter. Depending on the strength of these interactions, Sauls and Serene [1] have predicted the existence of modes with total angular momentum $J = 4$. The f-wave pairing is parameterized by,

$$x_3^{-1} = \frac{1}{\ln\left(\frac{T_{cf}}{T_c}\right)}, \tag{1}$$

where T_c is the transition temperature due to p-wave interactions and T_{cf} is the hypothetical transition temperature due to f-wave interactions that would occur in the absence of p-wave pairing. This form for the f-wave interaction parameter, x_3^{-1}, is chosen such that it is zero if they are negligible, and large but negative if they are significant and the f-wave pairing interaction is attractive. Sauls and Serene have shown that this f-wave pairing influences the frequency spectrum of collective modes, namely the real (+) and imaginary (-) squashing modes labelled as $J = 2^\pm$, each having a total angular momentum classification of 2; the plus and minus signs refer to the fact that different components of the order parameter are involved in each case, either real or imaginary. These squashing modes correspond to time-dependent, anisotropic, momentum-space deformations (or squashings) of the energy gap amplitude, $\delta\Delta^+(k, T)$. In the absence of quasiparticle interactions and f-wave pairing these modes have frequencies proportional to the gap [2] which we take from the weak-coupling plus model of Rainer and Serene [3]:

$$\Omega_{2^\pm} = a_\pm \Delta^+(T, P), \tag{2}$$

where $a_+ = \sqrt{8/5}$ and $a_- = \sqrt{12/5}$. In general, however, $a_\pm$ will have weak temperature and pressure dependences. Fig. 1 is a schematic of these mode frequencies as a function of temperature. To date, measurements of the imaginary squashing mode (ISQ) frequency have not been as precise as those of the real squashing mode

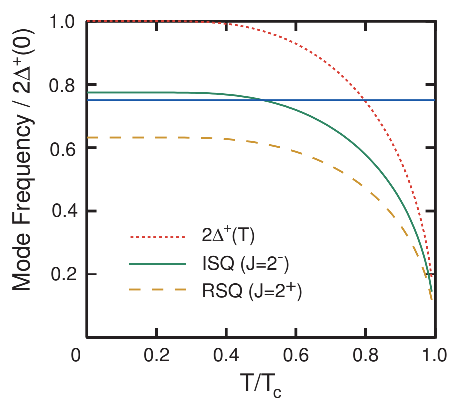

FIG. 1: (color online). Schematic of the collective mode spectrum for ^{3}He-B normalized to twice the value of the gap at zero temperature, $2\Delta^+(0)$. From low to high frequency we show the real squashing mode (dashed line), the imaginary squashing mode (solid line), and $2\Delta^+(T)$ (dotted line), the threshold for pair-breaking. The straight line is an example of a measurement frequency crossing the imaginary squashing mode and pair breaking.

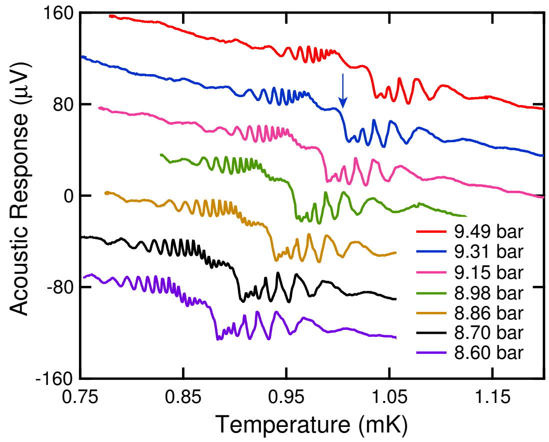

FIG. 2: (color online). Interference of transverse sound near the imaginary squashing mode using 99.9484 MHz sound at various pressures near 9 bar. Note that as the pressure is changed the frequency of the ISQ mode changes accordingly since the gap amplitude, Eq. 2, is approximately proportional to T_c. The cessation of wiggles and the bend in the acoustic impedance marks the location of the imaginary squashing mode, delineated by the arrow for 9.31 bar (see Fig.3).

(RSQ) and hence determination of x_3^{-1} from this mode has been inexact. We use a transverse acoustic cavity technique that takes advantage of the existence of transverse acoustic standing waves near the collective mode in order to measure presicely the ISQ mode frequency.

The ^{3}He sample is contained within a silver sample cell attached to a nuclear demagnetization refrigerator [4]. We used a lanthanum-diluted cerium magnesium nitrate paramagnetic salt thermometer [4] calibrated with respect to the superfluid phase diagram as given by Greywall [5]. Our AC-cut quartz transducers have coaxial electrode patterning and an overtone polish. An acoustic cavity, of spacing 48 microns, was formed from two transducers spaced by monodispersed polystyrene latex microspheres. We obtained a frequency resolution during experiments of $\Delta\nu/\nu \approx 2 \times 10^{-9}$ from 85 to 125 MHz (15$^{\text{th}}$ to 21$^{\text{st}}$ harmonics) using a CW, frequency-modulated, acoustic-impedance spectrometer [4].

Transverse sound measurements using acoustic impedance techniques in ^{3}He are reviewed by Halperin and Varoquaux [2]. The acoustic impedance is $Z = \rho/\omega q = \rho C$, where C is the complex phase velocity and $q = \omega/c + i\alpha$. Hence the acoustic impedance is simultaneously sensitive to changes in the (normal fluid) density, ρ; phase velocity, c; and attenuation, α. Transverse sound has been observed as a propagating mode in superfluid ^{3}He [6, 7] following the prediction of Moores and Sauls [8], the only liquid for which this has been demonstrated. As a result, the quartz transducer, which forms one side of a cavity, detects an impedance modulated by the sound wave reflected from the opposite cavity wall. The interference between outgoing and reflected waves gives rise to an oscillatory response in impedance as the velocity changes near the mode frequency, Fig. 2.

The acoustic cavity response as a function of temperature was measured for a sequence of pressures at fixed acoustic frequency, as shown for 99.9484 MHz near 9 bar in Fig. 2. As the temperature is lowered the ISQ frequency approaches, and then crosses, the fixed transverse sound frequency (where the straight line approaches the middle curve in Fig. 1). Correspondingly, according to the theoretical dispersion represented in Eq. 3, the phase velocity increases and transverse sound propagates with lower attenuation. In fact, it appears from this equation as though the velocity diverges at the crossing if viewed sufficiently far from the crossing itself. As the velocity increases so does the wavelength and for each half-wavelength that leaves the cavity there is one oscillation. As the ISQ mode frequency crosses the measurement frequency the order parameter mode resonantly absorbs acoustic energy and the interference pattern is extinguished. This corresponds to the sharp upward bend with decreasing temperature in the traces in Fig. 2. As the temperature is lowered further, transverse sound propagates again and the interference pattern reappears. Finally, as the temperature is lowered still further the transverse sound becomes highly attenuated. The details of the oscillations on the low temperature side of the mode and the acoustic impedance near the crossing point have not been observed previously and are not predicted by Eq. 3.

In order to determine the exact crossing temperature for the imaginary squashing mode with transverse sound, we plot, as in Fig. 3 at 9.31 bar, the temperature difference of sequential extrema in the interference pattern, δT, and extrapolate independently from both the high and low temperature sides of the ISQ. The temperature at which δT goes to zero, where the velocity diverges, marks this crossing point. The slopes of the lines in the figure are determined by the temperature depen-

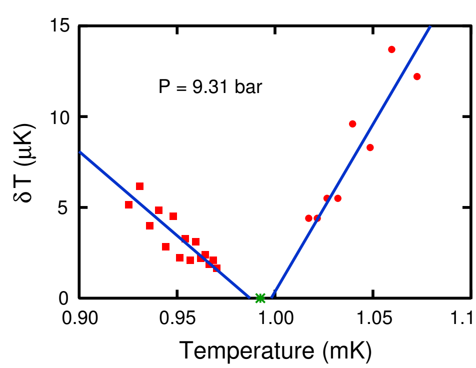

FIG. 3: (color online). Temperatures of maxima and minima in impedance for determining the ISQ mode crossing at 99.9484 MHz.

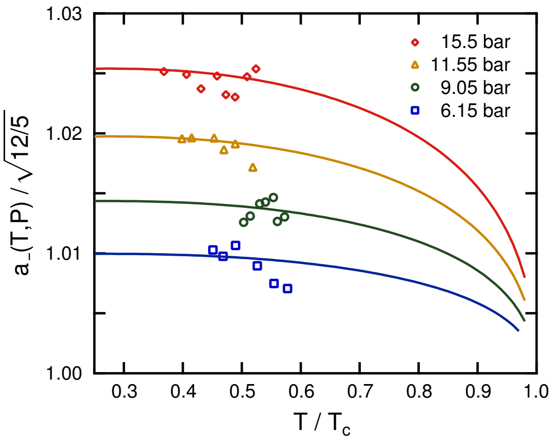

FIG. 4: (color online). Imaginary squashing mode crossings as determined by the oscillation period shift analysis of Fig. 3 corrected to be at constant pressures. The fits are to Eq. 4.

dence of the phase velocity of transverse sound. We find that they are different on the high and low temperature sides by about a factor of two. Our determinations of $a_-(T,P)/\sqrt{12/5}$ from the mode crossings are plotted for four fixed pressures in Fig. 4. The crossings are determined with a resolution of 5 to 15 μK, resulting in an uncertainty in a_- between 0.1% and 0.25%.

These data can be compared with the theory for transverse sound propagation in ^{3}He as given by Moores and Sauls [8]. They have shown that the dispersion of transverse sound is

$$\left(\frac{\omega}{qv_f}\right)^2 = \frac{F_1^s}{15}(1-\lambda) + \frac{2F_1^s}{75}\lambda \times \frac{\omega^2}{(\omega+i\Gamma)^2 - \Omega_{2^-}^2 - \frac{2}{5}q^2v_f^2}, \quad (3)$$

where v_f is the Fermi velocity, ω is the measurement frequency, $\Omega_{2^-} = a_-(T,P)\Delta^+(T,P)$ is the ISQ mode frequency, and $\lambda(\omega,T)$ is the Tsuneto function. $\Gamma(T)$ is the width of the mode, with an approximate form of $\Gamma(T) \simeq \Gamma_c\sqrt{T/T_c}e^{-\frac{\Delta(T)}{T}}$ and $\Gamma_c \sim 10^6 - 10^7$ Hz [8]. At temperatures low compared with T_c, the first term goes to zero like the normal fluid density, and the second term dominates. From Eq. 3, it can be seen that transverse sound couples off-resonantly to the ISQ mode at frequencies above $\Omega_{2^-}(T)$. Below the ISQ mode, owing to non-zero $\Gamma(T)$, transverse sound continues to propagate but highly attenuated. The condition for the crossing represented in Fig. 3 is shown by the vertical arrow in Fig. 2. This allows us to identify the position of the ISQ mode as a specific point along the trace of acoustic impedance. From Eq. 3 we see that there are no significant dispersion corrections to the mode frequency, a consequence of the fact that transverse sound propagates only through off-resonant coupling to the ISQ mode.

The influence of quasiparticle interactions, F_2^s, and f-wave pairing, x_3^{-1}, on the imaginary squashing mode frequency, was calculated by Sauls and Serene [1],

$$\Omega_{2^-}^2 - \frac{12}{5}\Delta^{+2} + \frac{3}{5}F_2^s(\Omega_{2^-}^2 - 4\Delta^{+2})\lambda + \frac{1}{4}x_3^{-1}\Omega_{2^-}^2(\Omega_{2^-}^2 - 4\Delta^{+2})\lambda/\Delta^{+2} = 0. \quad (4)$$

If $F_2^s = 0$ and $x_3^{-1} = 0$ then Eq. 4 reduces to Eq. 2 and one would expect the data in Fig. 4 to lie exactly at one, assuming correctness of the temperature scale [5]. The curved lines in Fig. 4 are fits to Eq. 4 using $\lambda(\omega,T)$ adapted to the weak-coupling plus [3] form of the gap, $\Delta^+(T,P)$. The only free parameter in these fits is taken to be x_3^{-1}, with F_2^s as an input [2]. In the top of Fig. 5 we plot $a_-(0,P)/\sqrt{12/5}$, the zero temperature intercepts of the fits in Fig. 4. We find that at zero pressure $a_-(0,0)/\sqrt{12/5} = 1$. This is expected if quasiparticle interactions become insignificant. There is reasonable evidence that both f-wave pairing interactions and F_2^s become small at low pressure [2]. However, any discrepancy between the Greywall temperature scale [5] and the absolute temperature scale will shift the data. Possible inaccuracy in the temperature scale was estimated by Greywall to be better than 1% [5]. Taking these extremes for the temperature scale as a constant factor gives the dashed lines in the top panel of Fig. 5. The level of spectroscopic precision is very high. Nonetheless, we can at most state that the *combined* effect of quasiparticle interactions, f-wave pairing, and inaccuracy in the temperature scale is negligible at $P = 0$.

In the lower portion of Fig. 5 we show the pressure dependence of the f-wave pairing strength, x_3^{-1}. These results hinge on the accuracy of F_2^s derived from measurements of the difference in first and zero sound velocities tabulated in Ref. 2. There is a reasonable consensus in published data at intermediate pressures near 10 bar and above; although less so at low pressure: *i.e.* $F_2^s = 0.17$, 0.34 and 0.5 at $P = 10$, 15 and 20 bar and $F_2^s \approx 0$ at $P = 0$. Taking previous work [2] into account we estimate that F_2^s could be larger but likely not more than $+0.25$ for any pressure affecting our determination of x_3^{-1} as shown by the dashed line in Fig. 5. In spite of these inaccuracies it is nonetheless clear that the f-wave pairing interaction becomes negative (attractive) with higher pressure.

The values of x_3^{-1} from our measurements can be compared with those from other techniques. Acoustic RSQ measurements [9] yield values of x_3^{-1} that roughly start at zero at zero pressure and increase to ~ -0.25 at 20 bar. These results depend on the Fermi liquid parameter F_2^a which is not well established. Analysis of the acoustic Faraday effect [7] yields a value of $x_3^{-1} = -0.375$ at 4.3 bar [10]. Meisel *et al.* [11] performed an analysis of longitudinal acoustic ISQ data to extract x_3^{-1}. They

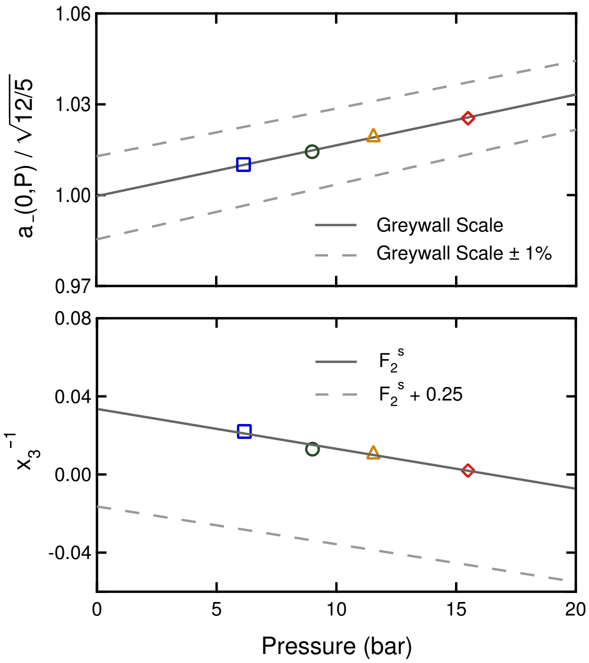

FIG. 5: (color online). Pressure dependence of (top) $a_-(0,P)/\sqrt{12/5}$ and (bottom) x_3^{-1} from the fits of Eq. 4 to the ISQ mode frequencies. The straight lines through the data are guides to the eye. The open symbols use the Greywall temperature scale [5]. The dashed lines (top) correspond to a $\pm 1\%$ change to this scale. The values of x_3^{-1} are obtained with F_2^s from Ref. [2]. The dashed line (bottom) shows the effect of adding 0.25 to F_2^s.

concluded that x_3^{-1} was ~ 0.2 at 0 bar, but decreased as the pressure increased becoming negative near 5 bar, then leveling off at ~ -0.2. The absolute values are generally much larger than what we infer from our data but we note that longitudinal sound measurements are inherently less precise since this sound mode couples so strongly to the ISQ mode. We find the pressure dependence of x_3^{-1} to be about a factor of six smaller than in these previous works independent of our estimated inaccuracy in the temperature scale or in F_2^s. We conclude that the predicted [1] $J = 4$ modes may exist but, if so, only very near or slightly below $2\Delta^+(T,P)$ [2]. Some of the modes in the 9-fold multiplet will couple to transverse sound and our transverse acoustic cavity technique should allow resolution of these collective modes at high frequencies very close to the particle-hole continuum. In addition, application of a magnetic field will be helpful for observation of $J = 4$ modes whose frequencies decrease by the Zeeman effect.

Fourier transform longitudinal acoustics experiments near the gap edge were performed at low pressure by Masuhara *et al.* [12]. They observed the onset of anomalously high attenuation at an energy (frequency) 4% lower than was expected from weak coupling BCS theory, $2\Delta_{BCS}$. Since it is unlikely that the superfluid ^{3}He-B order parameter is smaller than Δ_{BCS}, their results mean that either the temperature scale is imprecisely known or there is a new collective mode near the gap edge. Our measurements of the ISQ mode rule out that inaccuracy in temperature of this magnitude is a possible explanation and suggest that these authors have observed attenuation from higher order $J = 4$ order parameter collective modes.

In summary, we have made high precision measurements of an order parameter collective mode using interference of transverse sound in an acoustic cavity. Taking into account strong coupling effects we interpret our data to determine values of the f-wave pairing interaction strength that are much smaller than those from previous work. Despite inaccuracy in the parameters required for the analysis that reduce our resolution, we have set limits on the pressure dependence of the strength of f-wave interactions in superfluid ^{3}He. Increased accuracy in measurements of F_2^s will help to refine this conclusion. Lastly, the high resolution of the transverse acoustic cavity technique should make it possible to observe the predicted $J = 4$ order parameter collective modes.

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